Instructions: You are not permitted to use a calculator of any kind on any portion of this exam. You are not allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this exam. All devices must be turned off and stored away while you are in the examination room.

During this exam, any form of communication with any person other than the instructor or designated proctor is understood to be a violation of academic integrity.

No part of this exam may be removed from the examination room.

Read each question carefully. To receive full credit in the free response, you must:

- show legible, logical, and relevant justification which supports your final answer,
- use complete and correct mathematical notation,
- include proper units, where necessary, and
- give answers as exact values whenever possible.

You have ninety (90) minutes to complete this entire examination. Good luck!
Multiple Choice. On a piece of paper, write your answer selection to each of the following questions. You must clearly enumerate your responses. If no enumeration is provided, the grader will assume the ordering of your responses. Each question has only one correct answer, so if you indicate more than one answer, or leave a blank, the question is marked as incorrect. For this exam, there are fifteen (15) multiple choice questions worth three (3) points each for a total of forty-five (45) points.

1. Respond to this question with your exam version as indicated in the upper right-hand corner.
   *This question has no point value.*
   a. A
   b. B

2. Given that \( f'(2) = 0 \) and \( f''(2) = -1 \), then at \( x = 2 \), we’ve identified a ...
   a. local minimum.
   b. local maximum.
   c. not enough information.

3. Evaluate the expression: \( \frac{d}{dx} \left( \int_0^{x^2} \frac{1}{t} \, dt \right) \).
   a. \( \frac{1}{x^2} \)
   b. \( \ln|x^2| \)
   c. \( \frac{2}{x} \)

4. True or False: \( \int \cos^2 x \, dx = \frac{1}{3} \sin^3 x + C \).
   a. True
   b. False
5. Suppose we wish to maximize the product \( xy \) under the constraint that \( \sqrt{y} + x = 2 \). Which of the following is the single-variable function of \( y \) that will take the place of the two-variable objective?

   a. \( f(y) = (2 - \sqrt{y})y \)  
   b. \( f(y) = 2 - \sqrt{y} \)  
   c. \( f(y) = 2y - \sqrt{y} \)

6. If \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\), then for at least one value of \( c \) inside the interval \((a, b)\) ...

   a. \( f'(c) = 0 \)  
   b. \( f'(c) = \frac{f(b) - f(a)}{b - a} \)  
   c. \( f'(c) = c \)

7. Write the linearization for \( f(x) = 2 \cos x - 2x \) centered at \( c = 0 \).

   a. \( L(x) = 2x - 2 \)  
   b. \( L(x) = -2x \)  
   c. \( L(x) = 2 - 2x \)
8. L'Hôpital’s Rule may be applied to any limit.
   
a. True
b. False

9. Suppose that the radius of a circle increases by $\frac{1}{2}$ inch. Use a differential to approximate by about how much the circumference will increase. Select the best approximation.
   
a. $\pi$ in
b. $2\pi$ in
c. $\frac{3\pi}{2}$ in

10. Evaluate the limit: $\lim_{x \to \infty} \frac{e^{2x} - x}{3e^{2x}}$.
    
a. $\infty$  
b. 1  
c. $\frac{1}{3}$
11. Other than \(x = 0\), find the additional solution to the given equation.

\[
\int_0^x (2t - 2) \, dt = 0
\]

a. \(x = 2\) \hspace{1cm} b. \(x = 1\) \hspace{1cm} c. neither of these

12. Evaluate the indefinite integral: \(\int \frac{x^2 - 4}{x - 2} \, dx\).

a. \(\frac{1}{2}x^2 - 2x + C\) \hspace{1cm} b. \(\frac{1}{3}x^3 - 4x + C\) \hspace{1cm} c. \(\frac{1}{2}x^2 + 2x + C\)

13. Find \(y\) given that \(y' = e^t - 2\) and \(y(0) = 0\).

a. \(y(t) = e^t - 2t\) \hspace{1cm} b. \(y(t) = e^t - 2t - 1\) \hspace{1cm} c. \(y(t) = e^t - t^2 - 1\)
14. On which interval is \( f(x) = \frac{1}{3}x^3 - 4x \) concave down?

a. \((-\infty, 0)\)  

b. \((0, \infty)\)  

c. \((-\infty, \infty)\)

15. Use the graph of \( y = f(x) \) below with the given midpoint rectangles to calculate an approximation to the area under the curve on the interval \([0, 6]\).

![Graph of y = f(x) with midpoint rectangles]

a. 14 units\(^2\)  

b. 7 units\(^2\)  

c. 3.5 units\(^2\)

16. Suppose that \( \int_1^3 f(x) \, dx = 2 \). Find the value of \( \int_3^1 \frac{1}{2} f(x) \, dx \).

a. 1  

b. not enough info  

c. \(-1\)

This concludes the multiple choice portion of this exam.

Please take a moment to verify that you have sixteen (16) total responses, with the first response holding your exam version, Version A.
Free Response. Answer all questions clearly and legibly on blank paper. If your work is illegible, it may be marked incorrect. Additionally, correct answers without appropriate work shown will receive little or no credit. For each question, show specific mathematical notation, work, and any appropriate units. In addition, you may not use L'Hôpital's Rule on this portion of the exam. Before you begin, read the directions at the beginning of the exam. This portion of the exam is worth fifty-five (55) points.

1. (18 pts) Evaluate the following indefinite and definite integrals using basic techniques only. The use of any advanced techniques will result in no credit. You must show all work.

a. (4 pts) \( \int (x + \sqrt{x} - 2) \, dx \)

\[
\int (x + \sqrt{x} - 2) \, dx = \int (x + x^{\frac{1}{2}} - 2) \, dx = \frac{1}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}} - 2x + C
\]

b. (4 pts) \( \int \frac{2}{1 + 4y^2} \, dy \)  

HINT: \((4y^2) = (2y)^2\)

\[
\int \frac{2}{1 + 4y^2} \, dy = \int \frac{2}{1 + (2y)^2} \, dy = \arctan 2y + C
\]

c. (5 pts) \( \int_{0}^{\frac{\pi}{2}} \sec \theta \cos^2 \theta \, d\theta \)

\[
\int_{0}^{\frac{\pi}{2}} \sec \theta \cos^2 \theta \, d\theta = \int_{0}^{\frac{\pi}{2}} \frac{1}{\cos \theta} \cos^2 \theta \, d\theta = \int_{0}^{\frac{\pi}{2}} \cos \theta \, d\theta = \sin \theta \bigg|_{0}^{\frac{\pi}{2}} = \sin \frac{\pi}{2} - \sin 0 = 1
\]

d. (5 pts) \( \int_{-2}^{-1} \frac{1 + x}{x} \, dx \)

\[
\int_{-2}^{-1} \frac{1 + x}{x} \, dx = \int_{-2}^{-1} \left( \frac{1}{x} + 1 \right) \, dx = \ln |x| + x \bigg|_{-2}^{-1} = \cdots = 1 - \ln 2
\]
2. (10 pts) Suppose we want to maximize the area of a triangle but the base, \( b \), and the height, \( h \), must sum to five (5) meters. What is the maximum area of this triangle?

a. (1 pts) State the constraint equation.

\[ b + h = 5 \]

b. (2 pts) Write the objective function in terms of the variable \( b \). Distribute to simplify.

\[ h = 5 - b \implies A = \frac{1}{2}bh = \frac{1}{2}b(5 - b) \implies A(b) = \frac{5}{2}b - \frac{1}{2}b^2 \]

c. (2 pts) State the domain of the objective function found in part (b).

\[ 5 - b > 0 \implies 0 < b < 5 \text{ or } b \in (0, 5) \]

d. (5 pts) Apply the relevant calculus to solve the problem and write a summary statement in context that answers the question.

First, we find the derivative of \( A \), i.e. \( A'(b) = \frac{5}{2} - b \). Next, we will look for the critical points of \( A \). Since \( A \) is linear, we only need to look for \( A'(b) = 0 \), i.e. \( \frac{5}{2} - b = 0 \implies b = \frac{5}{2} \).

But does this maximize the area? (Second Derivative Test) Note that \( A''(b) = -1 \) which shows that \( A \) is concave down always. Hence, \( b = \frac{5}{2} \) is a local and global maximum. (First Derivative Test) Alternatively, \( A'(2) > 0 \) and \( A'(3) < 0 \), so \( b = \frac{5}{2} \) is a local and global maximum.

Finally, we can calculate this maximum area.

\[ b = \frac{5}{2} \implies h = 5 - \frac{5}{2} = \frac{5}{2} \implies A_{\text{max}} = \frac{1}{2} \cdot \frac{5}{2} \cdot \frac{5}{2} = \frac{25}{8} \text{ meters}^2 \]

Summary: Under the constraint, the maximum possible area of the triangle is \( \frac{25}{8} \) square meters.
3. **(9 pts)** Consider the following questions.

   a. **(6 pts)** Graphed is $y = e^x - \sin x$. Set up and evaluate a definite integral which yields the area of the shaded region. Simplify where possible and include units.

   
   \[
   A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (e^x - \sin x) \, dx = e^x + \cos x \bigg|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}
   \]

   \[
   = e^{\frac{\pi}{4}} + \cos \frac{\pi}{4} - \left( e^{-\frac{\pi}{4}} + \cos \left( -\frac{\pi}{4} \right) \right)
   \]

   \[
   = e^{\frac{\pi}{4}} + \frac{\sqrt{2}}{2} - e^{-\frac{\pi}{4}} - \frac{\sqrt{2}}{2}
   \]

   \[
   = e^{\frac{\pi}{4}} - e^{-\frac{\pi}{4}} \text{ units}^2
   \]

   b. **(3 pts)** Graphed is $y = -x^2 + \frac{7}{2}x - \frac{3}{2}$. Set up – **but do not evaluate** – a definite integral which would yield the area of the shaded region.

   \[
   A = \int_{\frac{1}{2}}^{3} \left( -x^2 + \frac{7}{2}x - \frac{3}{2} \right) \, dx
   \]
4. **(9 pts)** Let’s estimate the value of $\sqrt{26}$ using a linear approximation.

a. **(5 pts)** Write the linearization of $f(x) = \sqrt{x}$ centered at $c = 25$. Your ultimate response to this part should be a linear function.

$$f(25) = \sqrt{25} = 5 \quad f'(x) = \frac{1}{2\sqrt{x}} \implies f'(25) = \frac{1}{10}$$

$$L(x) = f(25) + f'(25)(x - 25) \implies L(x) = 5 + \frac{1}{10}(x - 25)$$

b. **(3 pts)** Use the linearization in part (a) to calculate an approximation for $\sqrt{26}$. Your ultimate response to this part should be a number.

$$\sqrt{26} = f(26) \approx L(26)$$

$$L(26) = 5 + \frac{1}{10}(26 - 25) = 5 + \frac{1}{10} = \frac{51}{10} = 5.1 \implies \sqrt{26} \approx 5.1$$

c. **(1 pts)** Suppose you’re given that $f''(25) \approx -0.002$. Is the estimate in part (b) an overestimate or underestimate? You do not need to justify your selection.

overestimate
5. (9 pts) Consider some unknown function \( y = f(x) \) whose domain is all real numbers. Its first derivative is \( f'(x) = -4x^2(3 + x) \) and its second derivative is \( f''(x) = -12x(2 + x) \).

a. (2 pts) Determine the intervals on which \( f \) is increasing and decreasing.

\[ f \text{ increases on } (-\infty, -3) \text{ and decreases on } (-3, 0), (0, \infty) \]

b. (2 pts) State the \( x \)-value(s) for any local extrema. Identify each as a local maximum or local minimum.

\[ f \text{ has a local maximum at } x = -3 \]

c. (2 pts) Determine the intervals on which \( f \) is concave up and concave down.

\[ f \text{ is concave down on } (-\infty, -2), (0, \infty) \text{ and concave up on } (-2, 0) \]

d. (2 pts) State the \( x \)-value(s) for the inflection point(s) of \( f \).

\[ f \text{ has inflection points at } x = -2 \text{ and } x = 0 \]

e. (1 pts) Based on the above, select the graph of the function \( f \). Simply respond with A or B.

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**A**

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**B**
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