Please refer to Gradescope for the grading rubric.

**Instructions:** You are not permitted to use a calculator of any kind on any portion of this exam. You are not allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this exam. All devices must be turned off and stored away while you are in the examination room.

During this exam, any form of communication with any person other than the instructor or designated proctor is understood to be a violation of academic integrity.

No part of this exam may be removed from the examination room.

*Read each question carefully.* To receive full credit in the free response, you must:

- show legible, logical, and relevant justification which supports your final answer,
- use complete and correct mathematical notation,
- include proper units, where necessary, and
- give answers as exact values whenever possible.

You have ninety (90) minutes to complete this entire examination. Good luck!
Multiple Choice. On a piece of paper, write your answer selection to each of the following questions. You must clearly enumerate your responses. If no enumeration is provided, the grader will assume the ordering of your responses. Each question has only one correct answer, so if you indicate more than one answer, or leave a blank, the question is marked as incorrect. For this exam, there are fifteen (15) multiple choice questions worth three (3) points each for a total of forty-five (45) points.

1. Respond to this question with your exam version as indicated in the upper right-hand corner. This question has no point value.
   a. A
   b. B

2. Solve the following equation for $x$: $2^x = 2^{1-x}$
   a. $x = 0$
   b. $x = 2$
   c. $x = \frac{1}{2}$

3. Complete the limit: $\lim_{x \to -\infty} \frac{2^x}{1 + \frac{1}{x}} = \cdots$
   a. $+2$
   b. $-2$

4. Find the equation of the line tangent to $y = x^2 - x$ at $x = 3$.
   a. $y - 6 = 5(x - 3)$
   b. $y - 6 = 6(x - 3)$
   c. $y - 6 = -\frac{1}{5}(x - 3)$
5. Evaluate the limit: \(\lim_{x \to 1^+} \frac{2x - 1}{2x - 2}\).

a. \(-\infty\)
b. \(+\infty\)

6. True or False: The graph of \(y = e^x - x\) has a horizontal tangent at \(x = 1\).

a. False
b. True

7. Evaluate the limit: \(\lim_{x \to \frac{\pi}{4}} \cos x \sin x\).

a. \(\frac{\sqrt{2}}{2}\)
b. 1
c. \(\frac{1}{2}\)
8. Find the **third derivative** of $y = x^3 - x + e^x$.

a. $y''' = 3x^2 - 1 + e^x$  
   b. $y''' = 6x - 1 + e^x$  
   c. $y''' = 6 + e^x$

9. Suppose that $\sqrt{2x - 3} \leq f(x) \leq \cos(x - 2)$ for all $x$-values close to $x = 2$. Calculate $\lim_{x \to 2} f(x)$.

a. $\lim_{x \to 2} f(x) = 0$  
   b. $\lim_{x \to 2} f(x) = 1$  
   c. $\lim_{x \to 2} f(x)$ D.N.E.

10. Evaluate the limit: $\lim_{x \to \infty} \frac{3x^2 - 2x + 1}{7x^2 + 4x - 3}$.

a. $\frac{3}{7}$  
   b. 0  
   c. $\infty$
11. Consider the following piecewise function where $a$ is an unknown constant.

$$f(x) = \begin{cases} x - a & x \geq 2 \\ x + a & x < 2 \end{cases}$$

For which of the following values of $a$ will $f$ be continuous at $x = 2$?

a. $a = 0$  

b. $a = 1$  

c. $a = 2$

12. Consider a function where $f(2) = 4$ and $f(-2) = 4$. The value of $f^{-1}(4)$ cannot be stated. Why?

a. $f$ is not continuous  

b. $f$ is not one-to-one  

c. $f$ is not differentiable

13. Use a derivative rule to evaluate the limit: $\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$.

a. 2  

b. $2(x + h) - 2x$  

c. $2x$
14. Suppose that \( \lim_{x \to 2^+} f(x) = 1 \) and \( \lim_{x \to 2^-} f(x) = 0 \). What type of discontinuity is present at \( x = 2 \)?

a. removable  

b. jump  

c. infinite

15. Consider \( y = \frac{x + 1}{x^2 - 1} \). A vertical asymptote exists at which of the following \( x \)-values?

a. \( x = -1 \) only  

b. \( x = 1 \) and \( x = -1 \)  

c. \( x = 1 \) only

16. Consider a rational function of the following form where \( k \) and \( n \) are positive whole numbers.

\[
y = \frac{x^k + x^{k-1} + \cdots + 1}{x^n + x^{n-1} + \cdots + 1}
\]

Under which of the following conditions will the function have an slant (oblique) asymptote?

a. \( k = n + 1 \)  

b. \( k = n \)  

c. \( k = n - 1 \)

This concludes the multiple choice portion of this exam.

Please take a moment to verify that you have responded to sixteen (16) questions, the first of which indicates your exam version, Version A.
Free Response. Answer all questions clearly and legibly on blank paper. If your work is illegible, it may be marked incorrect. Additionally, correct answers without appropriate work shown will receive little or no credit. For each question, show specific mathematical notation, work, and any appropriate units. In addition, you may not use L'Hôpital’s Rule on this portion of the exam. Before you begin, read the directions at the beginning of the exam. This portion of the exam is worth fifty-five (55) points.

1. **(16 pts)** Evaluate the following limits, showing all work. Remember that your notation is also graded. Absolutely no credit will be awarded for using L'Hôpital’s Rule.

   a. **(4 pts)** \[ \lim_{x \to 2} \frac{x - 2}{\sqrt{x - 1} - 1} \]

   \[ \lim_{x \to 2} \frac{x - 2}{\sqrt{x - 1} - 1} = \lim_{x \to 2} \frac{x - 2}{\sqrt{x - 1} - 1} \cdot \frac{\sqrt{x - 1} + 1}{\sqrt{x - 1} + 1} = \lim_{x \to 2} \frac{(x - 2)(\sqrt{x - 1} + 1)}{x - 2} = 2 \]

   b. **(4 pts)** \[ \lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} \]

   \[ \lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} = \lim_{x \to 4} \frac{(x - 4)(x + 4)}{(x - 4)(x + 3)} = \lim_{x \to 4} \frac{x + 4}{x + 3} = \frac{8}{7} \]

   c. **(4 pts)** \[ \lim_{x \to \infty} \frac{4x^4 + 2x + 1}{8x^5 + 6x + 2} \]

   \[ \lim_{x \to \infty} \frac{4x^4 + 2x + 1}{8x^5 + 6x + 2} = \lim_{x \to \infty} \frac{4x^4}{8x^5} + \frac{2x}{8x^5} + \frac{1}{8x^5} = \lim_{x \to \infty} \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = 0 + 0 + 0 = 0 \]

   d. **(4 pts)** \[ \lim_{x \to 1} \frac{1 - \frac{1}{x}}{x - 1} \]

   \[ \lim_{x \to 1} \frac{1 - \frac{1}{x}}{x - 1} = \lim_{x \to 1} \frac{x - 1}{x} \cdot \frac{1}{x - 1} = \lim_{x \to 1} \frac{1}{x} = 1 \]
2. (11 pts) Apply the limit definition of the derivative to find the derivative of \( f(x) = 2x^2 - x \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

You must show all work. The use of derivative rules will result in no credit.

\[
f'(x) = \lim_{h \to 0} \frac{2(x + h)^2 - (x + h) - (2x^2 - x)}{h}
\]

\[
= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - x - h - 2x^2 + x}{h}
\]

\[
= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - h - 2x^2 + x}{h}
\]

\[
= \lim_{h \to 0} \frac{4xh + 2h^2 - h}{h}
\]

\[
= \lim_{h \to 0} \frac{4x + 2h - 1}{h}
\]

\[
= 4x - 1
\]
3. (9 pts) Consider the function \( y = x + 2e^x \).

a. (2 pts) Find the derivative of \( y \) using derivative rules.

\[
y' = 1 + 2e^x
\]

b. (2 pts) Find the slope of the tangent line at \( x = \frac{1}{2} \).

\[
y'(\frac{1}{2}) = 1 + 2e^{\frac{1}{2}} = 1 + 2\sqrt{e}
\]

c. (2 pts) Find the slope of the normal line at \( x = 0 \).

\[
y'(0) = 1 + 2e^0 = 3 \implies m_N = -\frac{1}{3}
\]

d. (3 pts) Write the equation of the normal line at \( x = 0 \).

\[
x = 0 \implies y = 0 + 2e^0 = 2 \implies \text{point: (0, 2)}
\]

normal line: \( y - 2 = -\frac{1}{3}(x - 0) \iff y = -\frac{1}{3}x + 2 \)
4. (9 pts) Refer to the graph of the function $f$ below for each of the following questions. Assume that the graph continues outside of the region shown.

![Graph of function $f$](image)

**a. (1 pts)** Calculate $e^{f(0)} + \cos(f(0))$.

\[
f(0) = 0 \implies e^{f(0)} + \cos(f(0)) = e^0 + \cos 0 = 1 + 1 = 2
\]

**b. (1 pts)** Evaluate $\lim_{x \to -2} f(x)$.

\[
\lim_{x \to -2} f(x) \text{ D.N.E.}
\]

**c. (1 pts)** List all $x$-values where a jump discontinuity exists.

\[x = -2, x = 0\]

**d. (1 pts)** Evaluate $\lim_{x \to 0^-} f(x)$.

\[
\lim_{x \to 0^-} f(x) = 0
\]
e. **(1 pts)** Evaluate \( \lim_{x \to -4^+} f(x) \).

\[
\lim_{x \to -4^+} f(x) = -\infty
\]

f. **(2 pts)** Evaluate \( \lim_{x \to \infty} (-f(x)) \).

\[
\lim_{x \to \infty} (-f(x)) = -\lim_{x \to \infty} f(x) = -(3) = 3
\]

g. **(2 pts)** List all \( x \)-values where \( f \) is not differentiable.

\[x = -4, x = -2, x = -1, x = 0\]
5. **(10 pts)** Suppose an object in motion obeys the position function \( s(t) = \sqrt{t} \) feet where \( t \) is measured in seconds, \( t > 0 \). In the following, please include units.

a. **(4 pts)** Calculate the average velocity between \( t = 1 \) and \( t = 4 \) seconds.

\[
\text{v}_{\text{avg}} = \frac{s(4) - s(1)}{4 - 1} = \frac{\sqrt{4} - \sqrt{1}}{3} = \frac{1}{3} \text{ ft/s}
\]

b. **(6 pts)** Use the given limit formula to calculate the instantaneous velocity at \( t = 9 \) seconds.

\[
\text{v}_{\text{inst}} = \lim_{t \to 9} \frac{s(t) - s(9)}{t - 9}
\]

\[
\text{v}_{\text{inst}} = \lim_{t \to 9} \frac{\sqrt{t} - \sqrt{9}}{t - 9}
\]

\[
= \lim_{t \to 9} \frac{\sqrt{t} - 3}{t - 9} \cdot \frac{\sqrt{t} + 3}{\sqrt{t} + 3}
\]

\[
= \lim_{t \to 9} \frac{t - 9}{(t - 9)(\sqrt{t} + 3)}
\]

\[
= \lim_{t \to 9} \frac{1}{\sqrt{t} + 3}
\]

\[
= \frac{1}{\sqrt{9} + 3}
\]

\[
= \frac{1}{6} \text{ ft/s}
\]